Quantum Theory of Radiation

Introduction: In 1864 British physicist James Clerk Maxwell made suggestion that accelerated electric charges linked electric and magnetic disturbances that can travel indefinitely through space. If, the charges oscillate periodically, the disturbances are waves whose electric and magnetic components are perpendicular to each other and to the direction of motion.

James Clerk Maxwell (1831–1879)

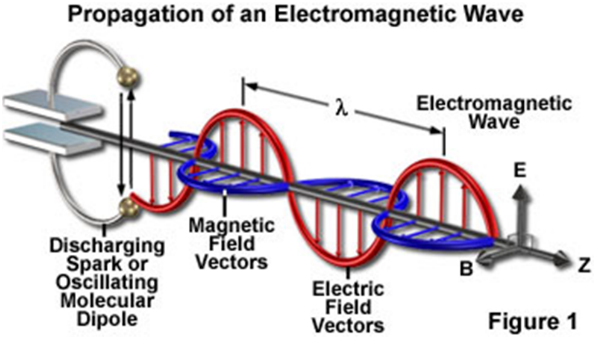


Figure 1

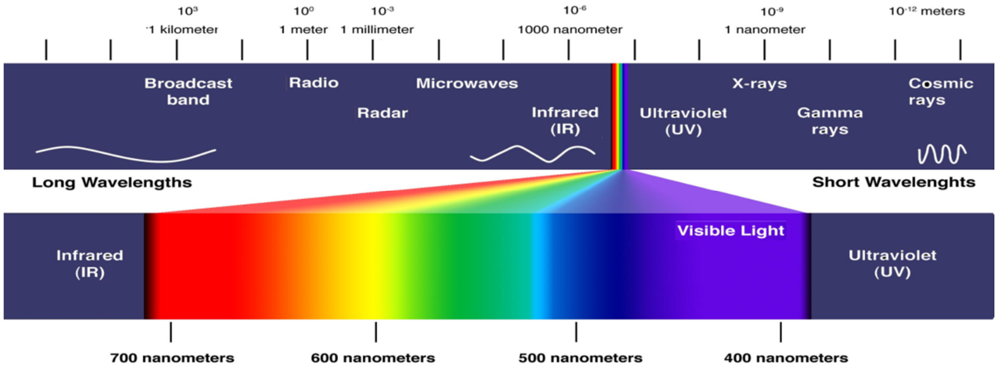
Thus electromagnetic wave nature of light was predicated by Maxwell and demonstrated experimentally by Hertz. This fact was further supported by explanation of the phenomena such as interference, diffraction, polarization on the basis of Huygens wave theory. However, the experimentally observed phenomena such as Compton effect, the spectrum of black body radiation and the characteristic spectra of atoms could not be explained on the basis of and Huygens wave theory. These phenomena, concerned with the emission and absorption of radiation by matter, indicated the existence of energy quanta.

Electromagnetic waves can be regarded as waves because under suitable circumstances they exhibit diffraction, interference, and polarization. Similarly, under other circumstances electromagnetic waves behave as though they consist of streams of particles.

Maxwell was able to show that the speed of electromagnetic wave in free space is given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.998\times 10^{8} \text{ m/s}.$$

where ε_0 is the electric permittivity of free space and μ_0 is the magnetic permeability. This is the same as light waves.

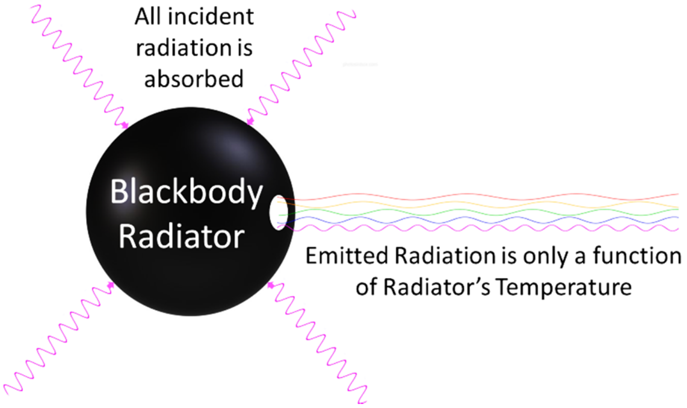


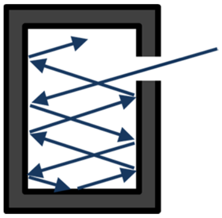
When a piece of iron rod is ignited it shows the following transition

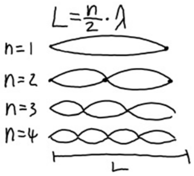
In fact, other frequencies to which our eyes do not respond are present as well. All objects radiate energy continuously whatever their temperature is. At room temperature most of the radiation is in the infrared part of the spectrum and hence invisible.

Black body radiation and Rayleigh-Jeans formula

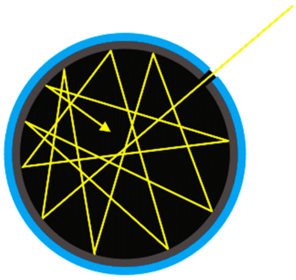
The ability of a body to radiate is closely related to its ability to absorb radiation. A body in thermal equilibrium with its surroundings absorbs energy at the same rate as it emits energy. An ideal body that absorbs all the radiation incident upon it, regardless of frequency is called a blackbody.



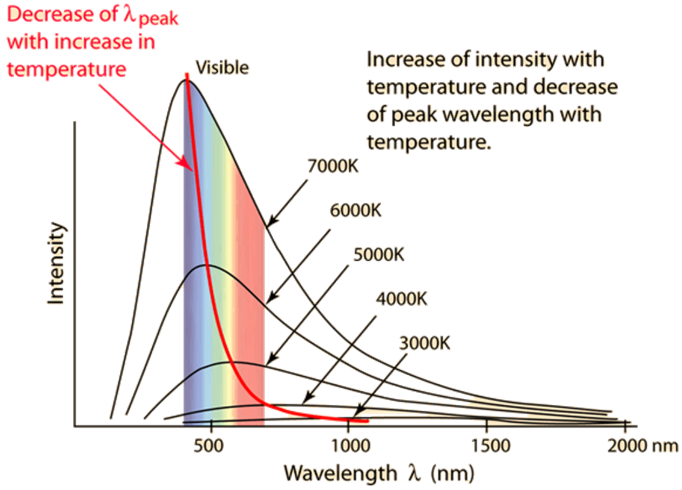




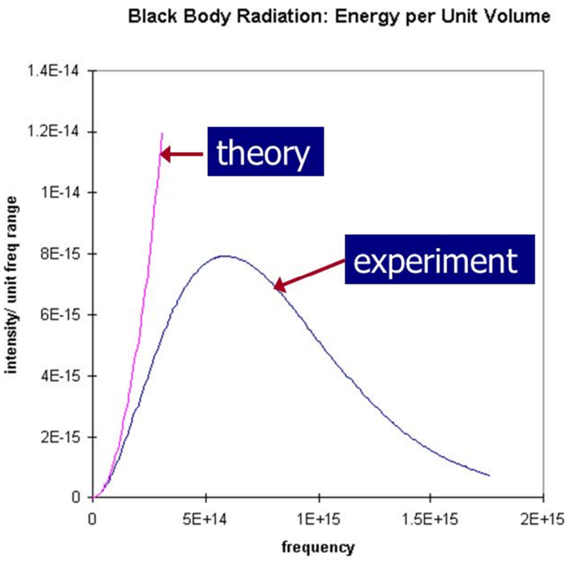
A hole in the wall of a hollow object is an excellent approximation of blackbody.

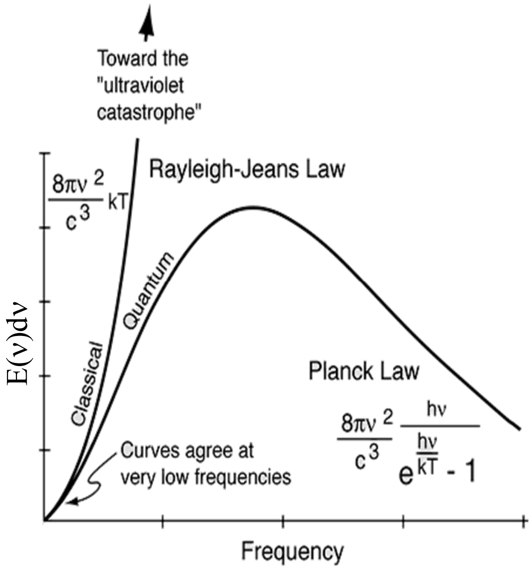


Conceptual Black Body



Electromagnetic radiation in the cavity whose walls are perfectly reflecting consists of standing waves that have nodes at the walls which restricts their possible wavelengths (**Quantization of EM waves**).





The total energy E(ν)dν of per unit volume in the cavity in the frequency interval from ν and (ν+dν) is $E(\nu)d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$ Rayleigh-Jeans Formula.

It shows, as $\nu\to\infty$, $E(\nu)d\nu\to\infty$. But, in reality as $\nu\to\infty$, $E(\nu)d\nu\to 0$.

This discrepancy is known as **ultraviolet catastrophe** of classical physics.

Planck's Radiation formula for the interpretation of quantum theory:

Planck consider a formula for the radiation like

$$E(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}d\nu$$

Where h is the Planck's constant and $h = 6.626\times10^{-34}$ J.S

At higher frequencies, $h\nu >> kT$ and therefore, $e^{\frac{h\nu}{kT}}\to\infty$. That is $E(\nu)d\nu\to 0$.

At lower frequencies, $h\nu << kT$ and $\frac{h\nu}{kT} << 1$. Therefore, $e^{\frac{h\nu}{kT}}\approx 1+\frac{h\nu}{kT}$.

Therefore, $E(\nu)d\nu \approx \frac{8\pi h}{c^3}\frac{\nu^3}{1+\frac{h\nu}{kT}-1}d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\frac{h\nu}{kT}}d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$ → Rayleigh-Jeans Formula.

At lower frequencies, Planck's Radiation formula →Rayleigh-Jeans formula and thus valid both for lower and higher frequency region and the ultraviolet catastrophe is removed.

He proposed the oscillator energy will be $E_n = nh\nu$, where n = 0, 1, 2, …. any integer.

That is, an oscillator emits radiation of frequency, ν, when it drops form one energy state to the next lower one and it jumps to the next higher state when it absorbs radiation of frequency, ν. Each discrete bundle of energy, $h\nu$ is called a **quantum** from a Latin word meaning "how much".

Average oscillator energy per standing wave therefore is not $\bar{\epsilon} = kT$, instead it is $\epsilon = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$, which leads to the Planck's radiation formula.